Likelihood Ratios

Likelihood ratios (LR) are used to express a change in odds. They are used most often in the realm of diagnosis. In this situation they combine test\(^1\) sensitivity and test specificity.

The positive likelihood ratio (+LR) gives the change in the odds of having a diagnosis in patients with a positive test. The change is in the form of a ratio, usually greater than 1. For example, a +LR of 10 would indicate a 10-fold increase in the odds of having a particular condition in a patient with a positive test result.

Note: The larger the +LR, the more informative the test. On the other hand, a +LR of 1.0 means the test is useless because the odds of having the condition have not changed after the test (a 1-fold increase in the odds means the odds have not changed).

The negative likelihood ratio (-LR) gives the change in the odds of having a diagnosis in patients with a negative test. The change is in the form of a ratio, usually less than 1. For example, a -LR of 0.1 would indicate a 10-fold decrease in the odds of having a condition in a patient with a negative test result. A -LR of 0.05 would be a 20-fold decrease in the odds of the condition.

Note: The smaller the -LR, the more informative the test. Of course, a -LR of 1.0 still means the test is useless because the odds of having the condition have not changed after the test (a 1-fold decrease in the odds means the odds have not changed).

Here’s the problem

Odds and likelihood ratios are not intuitive and are not easy to relate to.

For example, a 10 fold increase in odds that a patient has a disc herniation doesn’t really answer the question: “But, doc, what are the chances [i.e., probability] that I have a disc herniation?”

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1 In this context, test is used broadly and can refer to almost any clue used to make a diagnosis: symptoms (e.g., chest pain, nausea, paresthesia), particular features of the patient’s presentation (e.g., made worse by standing or sitting, has an episodic or chronic course, lasts 5 minutes or 5 hours), procedures (e.g., palpation, ortho or neurological tests), or ancillary tests (e.g., x-ray, CBC, EKG).
An LR of 10 does \textit{NOT} denote a 10 fold increase in probability! The increase in probability is usually a great deal less than that. In many cases, it would represent about a 45\% increase in probability, not a 1000 \% increase.

LRs can be used to project the change in the probability of having a particular condition from what we first thought (i.e., the pre-test probability) to the probability \textit{after the test results are interpreted} (i.e., the post test probability).

But to do this a “translation” step must be taken.

\textbf{Here’s the solution}

Here are four simple ways to calculate/estimate post test probability using LRs.

1. Limit yourself to a \textit{qualitative} sense of the change in probability.
2. Make a rough estimate of the post test probability.
3. Use a likelihood ratio calculator.
4. Use a nomogram.

\begin{table}[h]
\centering
\begin{tabular}{|l|}
\hline
\textbf{1 \ Get a qualitative sense} \\
\hline
\textbf{Get a qualitative sense} \\
\hline
A relatively high likelihood ratio of 10 or greater will result in a \textit{large} and \textit{significant} increase in the probability of a disease, given a positive test. \\
A LR of 5 will \textit{moderately} increase the probability of a disease, given a positive test. \\
A LR of 2 only increases the probability a \textit{small amount}. \\
A relatively low likelihood ratio (0.1) will significantly decrease the probability of a disease, given a negative test. \\
A LR of 1.0 means that the test is not capable of changing the post-test probability either up or down and so the test is not worth doing! \\
\hline
\end{tabular}
\end{table}
2 Make a rough estimate.

You can learn a few “LR to probability” conversions that can be used as reference points. These references points can then be used to make a rough estimate. Likelihood ratios of 2, 5, and 10 are associated with an increase in the probability of disease in the presence of a positive test, as follows:

+LR = 2 increases the probability of the disease by \(-15\) percent
+LR = 5 increases the probability of the disease by \(-30\) percent
+LR = 10 increases the probability of the disease by \(-45\) percent

Negative LRs of 0.5, 0.2, and 0.1 are associated with a decrease in the probability of a disease in the presence of a negative test, as follows:

-LR = 0.5 decreases the probability of the disease by \(-15\) percent
-LR = 0.2 decreases the probability of the disease by \(-30\) percent
-LR = 0.1 decreases the probability of the disease by \(-45\) percent

Note: these are only rough estimates. They work best when the pre-test probability hovers around 50%. They do not work when there is at a very low probability (approaching 10% or less) or very high probability (approaching 90% or more).

3 Use a likelihood ratio calculator.

There are a number of sites on the Web that have calculators which allow you to simply plug in your estimated prevalence (which, in this case, is essentially the same thing as pre-test probability) and a known likelihood ratio. The resulting increase or decrease in post-test probability will be calculated for you. An easy one to use is at EBM and Decision Tools by Alan Schwartz, Diagnostic Test Calculator at araw.mede.uic.edu/~alansz/tools.html. Another one is at the Centre for Evidence-Based Medicine http://www.cebm.net/index.aspx?o=1161
Use a nomogram.

Easy to use

Find the pretest probability (or prevalence) on the left vertical line. Next find the LR on the middle vertical line. Place a ruler or other straight edge connecting the two and follow it to the predicted posttest probability on the right vertical line.
**Likelihood ratios in other settings**

Likelihood ratios can be used in other situations. For example, they can be used in prognosis studies to express the likelihood of a bad outcome.

Example

In a population of low back pain patients with an estimated 11% pre-test probability of being out of work at one year due to pain, a high score on the Roland Morris Questionnaire (LR 2.1) would raise the pre-test probability from 11% to a post-test probability of 20%. If the pre-test test probability for a poor outcome is at the higher end of the predicted range at 33%, the post probability would increase to 51%. (Chou 2010)

**What if the LR is not reported?**

Sometimes one only has the reported sensitivity and specificity of a test. With this information it is very easy to calculate both the +LR and the -LR.

Positive LR = \[
\frac{\text{sensitivity}}{1 - \text{specificity}}
\]

Negative LR = \[
\frac{1 - \text{sensitivity}}{\text{specificity}}
\]

Let’s try a few:

1. Sensitivity: 90%, Specificity: 25%  Positive LR = ?  Negative LR = ?
2. Sensitivity: 40%, Specificity: 95%  Positive LR = ?  Negative LR = ?
3. Sensitivity: 95%, Specificity: 75%  Positive LR = ?  Negative LR = ?
4. Sensitivity: 67%, Specificity: 89%  Positive LR = ?  Negative LR = ?

(Answers on the last page.)

**Bottom line**

LRs are used to express how good a test is at increasing or decreasing the chances that someone has a particular disease or condition. Although LRs denote a change in odds, they can be used to determine a change in probability or the probability of having a condition.

Remember: an LR *always links* a test or risk factor to a particular condition or interpretation. For example, to express that a resting tremor has an LR of 23 *doesn’t mean anything!* Does it increase the odds of a nervous system disease in general? Of thyroid disease? Of Parkinson’s disease?
A few warnings are in order when considering LRs from the research literature.

1. LRs for the same test can vary from study to study and are only as valid as the quality of research which reports them. LRs reported in high quality systematic reviews, in general, may be more trustworthy than LRs generated by a single study.

2. Most LRs in the musculoskeletal domain reflect how effective tests are in a surgical or specialty clinic environment (e.g., patients referred to an orthopedist or neurologist). Chiropractors, however, see a broader mix of patients. These include patients with mild or moderate presentations of these conditions along with patients whose presentations are severe enough that they might end up in surgery or a specialist’s office. Generally, be aware that many of the tests may have lower LRs in a chiropractic or primary care setting.

QUICK REFERENCE TABLE

Interpreting LRs: Very useful LRs: > 10 and < .10

Note: + LR reflects the results of a positive test; -LR reflects the results of a negative test.

<table>
<thead>
<tr>
<th>Estimated % change in probability</th>
<th>+LR Power to RULE IN</th>
<th>SHIFT IN POST-TEST PROBABILITY</th>
<th>-LR Power to RULE OUT</th>
<th>Estimated % change in probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 = 45% ↑</td>
<td>10</td>
<td>← LARGE →</td>
<td>&lt; 0.1</td>
<td>0.1 = 45% ↓</td>
</tr>
<tr>
<td>5 = 30% ↑</td>
<td>5-10</td>
<td>← MODERATE →</td>
<td>.1-.2</td>
<td>0.2 = 30% ↓</td>
</tr>
<tr>
<td>2 = 15% ↑</td>
<td>2-5</td>
<td>← SMALL → (but sometimes important)</td>
<td>.2-.5</td>
<td>0.5 = 15% ↓</td>
</tr>
</tbody>
</table>

Note: These estimates apply best when pre-test probability is 50%; are smaller with higher & lower pre-test probabilities; do not apply when pre-test probability is <10% or >90%.

Ron LeFebvre, DC, MA
Reviewed by Mitch Haas, DC MA
2/1/11

References


Answers to calculation exercise

<table>
<thead>
<tr>
<th></th>
<th>+LR</th>
<th>-LR</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>0.4</td>
<td>small effect</td>
</tr>
<tr>
<td>2</td>
<td>8.0</td>
<td>0.6</td>
<td>poor</td>
</tr>
<tr>
<td>3</td>
<td>3.8</td>
<td>0.7</td>
<td>small to moderate</td>
</tr>
<tr>
<td>4</td>
<td>6.0</td>
<td>0.4</td>
<td>moderate</td>
</tr>
</tbody>
</table>